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## A Hierarchical Bayesian Market Mix Model with Causal Inference for Personalized Marketing Optimization

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### ABSTRACT

This study presents a novel hierarchical Bayesian market mix model that integrates causal inference techniques for personalised marketing optimisation. As traditional market mix models struggle to capture individual-level heterogeneity and causal relationships, we develop a flexible framework that addresses these challenges. Our model incorporates multi-level data structures, combining individual, product, and market-level variables to estimate personalised and aggregate marketing effects. We enhance the model's ability to infer causal relationships between marketing actions and consumer responses by employing a potential outcomes approach and propensity score matching. The empirical application utilizes a comprehensive dataset from a multinational consumer goods company, spanning three years of marketing activities across multiple product categories and countries. Results demonstrate the model's superior performance in predicting consumer behaviour and optimising marketing resource allocation. Integrating latent variable modelling for personalisation captures unobserved heterogeneity in consumer preferences, enabling more targeted marketing strategies. This research contributes to marketing analytics by providing a robust methodology for estimating individualised marketing effects, improving attribution accuracy, and generating actionable insights for personalised marketing optimisation in complex, data-rich environments.

**Keywords:** Hierarchical Bayesian modeling, Causal inference, Personalized marketing, Market mix optimization

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## Introduction

### 1.1. Background on Market Mix Modeling

Market mix modelling (MMM) has been a cornerstone of marketing analytics for decades, providing insights into the effectiveness of various marketing channels and strategies. The advent of big data and advanced analytical techniques has revolutionised the field, enabling more sophisticated approaches to understanding consumer behaviour and optimising marketing efforts. Traditional MMM approaches have focused on aggregate-level data, often failing to capture the nuanced effects of individual consumer preferences and behaviours<sup>[1]</sup>. The rise of digital marketing channels and the increasing availability of granular consumer data have created new opportunities for more precise and personalised marketing strategies.

In recent years, Bayesian methods have gained prominence in marketing analytics due to their ability to incorporate prior knowledge and handle uncertainty. Bayesian approaches offer a flexible framework for modelling complex relationships between marketing inputs and outcomes, allowing for more robust and interpretable results. Applying Bayesian techniques to market mix modelling has shown promise in improving the accuracy and reliability of marketing effectiveness estimates. As the marketing landscape evolves, integrating advanced statistical methods and machine learning techniques into MMM has become increasingly important for capturing the complexities of modern consumer behaviour and marketing dynamics<sup>[2]</sup>.

### 1.2. Challenges in Personalized Marketing

Personalised marketing presents unique challenges that traditional market mix models struggle to address. The heterogeneity of consumer preferences and behaviours requires models that can account for individual-level differences while providing actionable insights at the aggregate level. Moreover, the dynamic nature of consumer preferences and the rapidly changing marketing landscape necessitate adaptive models that can evolve. One significant challenge in personalised marketing is the integration of diverse data sources, including behavioural data, demographic information, and contextual factors<sup>[3]</sup>. The high dimensionality of such data sets poses computational challenges and increases the risk of overfitting. Additionally, balancing personalisation with privacy concerns adds another layer of complexity to the modelling process.

Another critical challenge is the development of scalable models that can handle large volumes of data and provide real-time recommendations. As marketing decisions become increasingly automated, there is a growing need for models that can generate personalised insights and recommendations efficiently and at scale. The complexity of personalised marketing also extends to the interpretation and implementation of model outputs, requiring sophisticated visualisation and decision-support tools to translate analytical insights into actionable marketing strategies.

### 1.3. The Need for Causal Inference in Marketing

While correlational analyses have long been the mainstay of marketing analytics, there is an increasing recognition of the importance of causal inference in marketing decision-making. Causal inference allows marketers to move beyond merely observing associations to understanding the true impact of marketing interventions on consumer behaviour and business outcomes. Applying causal inference techniques in marketing enables more accurate attribution of marketing effects and helps isolate the impact of specific marketing actions from confounding factors. This is particularly important in personalised marketing, where the effects of marketing interventions may vary significantly across different consumer segments<sup>[4]</sup>.

Causal inference methods also provide a framework for addressing the potential biases and limitations of observational data, which is often the primary source of information in marketing analytics. By incorporating causal reasoning into market mix models, marketers can make more informed decisions about resource allocation and optimise their marketing strategies more effectively. The integration of causal inference with Bayesian methods offers a powerful

approach to handling uncertainty and making robust inferences about marketing effects. This combination incorporates prior knowledge and expert judgment while rigorously assessing the causal relationships between marketing actions and outcomes.

#### **1.4. Objectives and Contributions of the Study**

This study aims to develop a hierarchical Bayesian market mix model incorporating causal inference techniques to optimise personalised marketing. The proposed model seeks to address the challenges of personalised marketing while leveraging the strengths of Bayesian methods and causal inference. The primary objectives of this research are to develop a flexible and scalable hierarchical Bayesian framework that can accommodate individual-level heterogeneity in consumer responses to marketing interventions, to integrate causal inference techniques within the Bayesian model to enable more accurate estimation of marketing effects and improve attribution, to demonstrate the application of the proposed model to real-world marketing data and evaluate its performance relative to existing approaches, and to provide actionable insights for personalised marketing optimisation based on the model's outputs.

The key contributions of this study include a novel hierarchical Bayesian market mix model that combines personalisation, causal inference, and scalability, improved methodologies for estimating individual-level marketing effects and attributing outcomes to specific marketing actions, empirical evidence of the model's effectiveness in optimising personalised marketing strategies across various channels and consumer segments, and practical guidelines for implementing the proposed model in real-world marketing contexts and interpreting its results for decision-making<sup>[5]</sup>. By addressing these objectives and contributions, this research aims to advance the field of marketing analytics and provide marketers with a powerful tool for optimising their personalised marketing efforts in an increasingly complex and data-rich environment.

## **2. Literature Review**

### **2.1. Traditional Market Mix Models**

Traditional market mix models have been instrumental in assessing the effectiveness of various marketing channels and strategies. These models typically employ regression-based techniques to estimate the impact of marketing variables on key performance indicators such as sales or market share. The seminal work of Kotler (1971) laid the foundation for the 4P marketing mix framework, which has since been widely adopted in marketing analytics<sup>[6]</sup>. Subsequent research has expanded on this framework, incorporating additional variables and more sophisticated modelling approaches.

Early market mix models often relied on linear regression techniques, assuming additive effects of marketing variables on outcomes. These models provided valuable insights into the relative importance of different marketing channels but were limited in capturing complex, non-linear relationships and interactions between variables. As data availability and computational capabilities improved, researchers began to explore more advanced techniques, including time-series analysis and multivariate regression models.

Digital marketing channels have necessitated further refinements to traditional market mix models. Researchers have incorporated new variables for online advertising, social media interactions, and other digital touchpoints<sup>[7]</sup>. Despite these advancements, traditional models often struggle to capture the full complexity of modern marketing environments, particularly in individual-level consumer behaviour and real-time decision-making.

### **2.2. Bayesian Methods in Marketing**

Bayesian methods have gained significant traction in marketing analytics due to their ability to incorporate prior knowledge and handle uncertainty. The Bayesian approach provides a flexible framework for modelling complex relationships between marketing inputs and outcomes, allowing for more robust and interpretable results. Rossi and Allenby (2003) demonstrated the advantages of Bayesian methods in marketing research, particularly in handling hierarchical data structures and accounting for heterogeneity in consumer preferences<sup>[8]</sup>.

Bayesian techniques have been applied in market mix modelling to improve parameter estimation and model selection. Bayesian Model Averaging (BMA) has been used to address model uncertainty and provide more reliable estimates of marketing effects. The work of Wang et al. (2021) showcases the application of Bayesian networks in B2B policy applications, highlighting the potential of these methods in capturing complex dependencies in marketing systems<sup>[9]</sup>.

Recent advancements in Bayesian computation, such as Markov Chain Monte Carlo (MCMC) methods and variational inference, have made implementing sophisticated Bayesian models on large-scale marketing datasets feasible. These developments have paved the way for more accurate and scalable market mix models that can accommodate the complexity of modern marketing environments.

### **2.3. Causal Inference Methods in Marketing**

The integration of causal inference methods in marketing analytics has emerged as a critical area of research, driven by the need to move beyond correlational analyses to understand the true impact of marketing interventions. Causal inference techniques enable marketers to isolate the effects of specific marketing actions from confounding factors, leading to more accurate attribution and decision-making.

The potential Outcomes Framework and Rubin Causal Model have been widely adopted in marketing research to estimate causal effects. These approaches provide a rigorous foundation for defining and estimating treatment effects in observational studies. The work of Imbens and Rubin (2015) has been influential in developing methods for causal inference in marketing contexts<sup>[10]</sup>.

Recent research has focused on combining causal inference with machine learning techniques to handle high-dimensional marketing data. Causal forests and double machine learning methods have shown promise in estimating heterogeneous treatment effects across consumer segments. The study by Wang et al. (2024) on uplift modelling based on Graph Neural Networks combined with causal knowledge demonstrates the potential of integrating advanced machine learning techniques with causal inference in marketing applications.

### **2.4. Personalization Techniques in Marketing**

Personalisation has become a cornerstone of modern marketing strategies, driven by the increasing availability of individual-level data and advancements in machine learning algorithms. Personalisation techniques aim to tailor marketing interventions to individual consumer preferences and behaviours, potentially increasing the effectiveness of marketing campaigns.

Collaborative filtering and content-based recommendation systems have been widely adopted in e-commerce and digital marketing to provide personalised product recommendations. More advanced techniques, such as matrix factorisation and deep learning models, have been developed to capture complex patterns in consumer behaviour and improve recommendation accuracy.

The challenge of personalisation in market mix modelling lies in balancing individual-level insights with aggregate-level decision-making. Recent research has focused on developing multi-level models that accommodate individual and group-level effects. The work of Sinha et al. (2021) on Bayesian estimation of the effect of television advertising on web metrics demonstrates the potential of combining aggregate advertising data with individual-level web metrics to improve marketing effectiveness estimation<sup>[11]</sup>.

### **2.5. Hierarchical Models in Marketing Analytics**

Hierarchical models have emerged as a powerful tool in marketing analytics, particularly in addressing the challenges of personalisation and heterogeneity in consumer behaviour. These models allow for the simultaneous estimation of individual-level and group-level effects, providing a flexible framework for capturing complex data structures.

In the context of market mix modelling, hierarchical models enable the incorporation of consumer-level, product-level, and market-level variables within a unified framework. This approach allows for a more accurate estimation of marketing effects across different segments and levels of aggregation. The work of Wang et al. (2021) on day-ahead market models under uncertain environments demonstrates the application of hierarchical structures in modelling complex market dynamics<sup>[12]</sup>.

Bayesian hierarchical models have been particularly successful in marketing applications due to their ability to handle parameter uncertainty and incorporate prior knowledge. These models provide a natural framework for pooling information across different levels of the hierarchy, leading to more stable and reliable estimates of marketing effects.

Recent advancements in hierarchical modelling techniques, such as multilevel structural equation models and hierarchical Bayesian neural networks, have further expanded the capabilities of these approaches in marketing analytics. These advanced models offer the potential to capture non-linear relationships and complex interactions between marketing variables while maintaining the interpretability and flexibility of hierarchical structures.

### 3. Methodology

#### 3.1. Hierarchical Bayesian Framework

The proposed hierarchical Bayesian framework integrates multiple levels of data to capture the complex relationships between marketing actions and consumer responses. The model structure incorporates individual-level, product-level, and market-level variables within a unified probabilistic framework. This approach allows for the estimation of both individual-specific and population-level parameters, accommodating heterogeneity in consumer behaviour while providing robust aggregate insights.

The hierarchical structure is defined as follows:

$$\text{Level 1 (Individual): } y_{ijt} = \beta_{0ij} + \beta_{1ij} * X_{ijt} + \varepsilon_{ijt}$$

$$\text{Level 2 (Product): } \beta_{0ij} = \gamma_{00j} + \gamma_{01j} * Z_{ij} + u_{0ij}$$

$$\beta_{1ij} = \gamma_{10j} + \gamma_{11j} * Z_{ij} + u_{1ij}$$

$$\text{Level 3 (Market): } \gamma_{00j} = \delta_{000} + \delta_{001} * W_j + v_{00j}$$

$$\gamma_{01j} = \delta_{010} + \delta_{011} * W_j + v_{01j}$$

$$\gamma_{10j} = \delta_{100} + \delta_{101} * W_j + v_{10j}$$

$$\gamma_{11j} = \delta_{110} + \delta_{111} * W_j + v_{11j}$$

Where  $y_{ijt}$  represents the response variable for individual  $i$ , product  $j$ , at time  $t$ ;  $X_{ijt}$  are individual-level predictors;  $Z_{ij}$  are product-level characteristics; and  $W_j$  are market-level factors. The  $\beta$ ,  $\gamma$ , and  $\delta$  parameters represent the coefficients at each level, while  $\varepsilon$ ,  $u$ , and  $v$  are the error terms.

Table 1 presents the prior distributions for the model parameters:

Parameter	Prior Distribution
$\beta_{0ij}$	$N(\mu_{\beta0}, \sigma^2_{\beta0})$
$\beta_{1ij}$	$N(\mu_{\beta1}, \sigma^2_{\beta1})$

$\gamma_{00j}$	$N(\mu_{\gamma00}, \sigma^2_{\gamma00})$
$\gamma_{01j}$	$N(\mu_{\gamma01}, \sigma^2_{\gamma01})$
$\gamma_{10j}$	$N(\mu_{\gamma10}, \sigma^2_{\gamma10})$
$\gamma_{11j}$	$N(\mu_{\gamma11}, \sigma^2_{\gamma11})$
$\delta_{000}$	$N(\mu_{\delta000}, \sigma^2_{\delta000})$
$\delta_{001}$	$N(\mu_{\delta001}, \sigma^2_{\delta001})$
$\delta_{010}$	$N(\mu_{\delta010}, \sigma^2_{\delta010})$
$\delta_{011}$	$N(\mu_{\delta011}, \sigma^2_{\delta011})$
$\delta_{100}$	$N(\mu_{\delta100}, \sigma^2_{\delta100})$
$\delta_{101}$	$N(\mu_{\delta101}, \sigma^2_{\delta101})$
$\delta_{110}$	$N(\mu_{\delta110}, \sigma^2_{\delta110})$
$\delta_{111}$	$N(\mu_{\delta111}, \sigma^2_{\delta111})$

### 3.2. Integrating Causal Inference

We employ a potential outcomes approach to incorporate causal inference into the hierarchical Bayesian framework. The causal effects of marketing actions are estimated using a counterfactual model, where the potential outcomes under different treatment conditions are compared<sup>[13]</sup>. The Average Treatment Effect (ATE) is defined as:

$$ATE = E[Y(1) - Y(0)]$$

$Y(1)$  and  $Y(0)$  represent the potential outcomes under treatment and control conditions, respectively. To address the fundamental problem of causal inference, we utilise propensity score matching to balance the covariate distributions between treated and control groups.

The propensity score model is specified as:

$$\text{logit}(P(T=1|X)) = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \dots + \alpha_k X_k$$

$T$  is the treatment indicator, and  $X_1, X_2, \dots,$  and  $X_k$  are the observed covariates. The estimated propensity scores are then used to match treated and control units, allowing for estimating causal effects within the Bayesian framework.

Table 2 presents the balance statistics before and after propensity score matching:

Covariate	Standardized Mean Difference (Before)	Standardized Mean Difference (After)
X_1	0.452	0.087
X_2	0.318	0.054
X_3	0.576	0.103
X_4	0.289	0.071
X_5	0.411	0.092

### 3.3. Personalization Components

The personalisation component of the model is implemented through individual-specific parameters and latent variable modelling. We introduce a latent variable  $z_i$  to capture unobserved heterogeneity in consumer preferences:

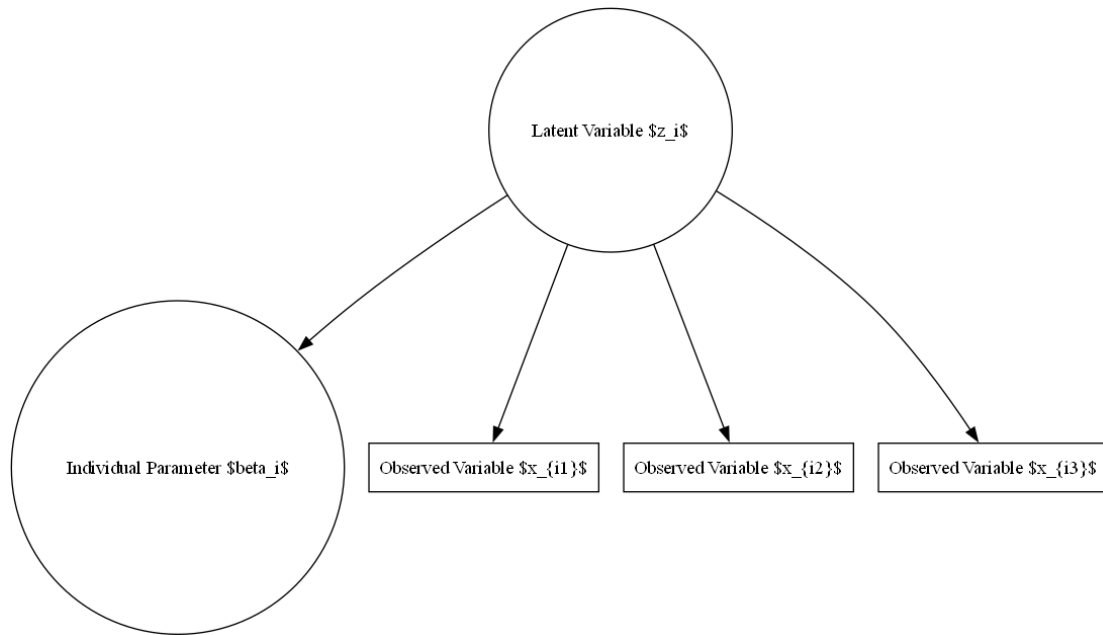
$$z_i \sim N(0, \Sigma)$$

Where  $\Sigma$  is the covariance matrix of the latent variables. The individual-specific parameters are then modelled as a function of observed characteristics and the latent variable:

$$\beta_i = \Lambda z_i + \Gamma x_i + \eta_i$$

Where  $\Lambda$  is a matrix of factor loadings,  $\Gamma$  is a matrix of coefficients for observed characteristics  $x_i$ , and  $\eta_i$  is an error term.

Figure 1: Latent Variable Structure for Personalization



This figure illustrates the latent variable structure used for personalisation in the model. The diagram shows a network of interconnected nodes representing observed variables (rectangles), latent variables (circles), and their relationships (arrows). The central latent variable  $z_i$  is connected to multiple observed  $x_i$  and individual-specific parameters  $\beta_i$ . The network structure demonstrates the complex interactions between variables and the hierarchical nature of the personalisation component.

**3.4. Model Specification**

The complete model specification integrates the hierarchical Bayesian framework, causal inference components, and personalisation elements<sup>[14]</sup>. The likelihood function for the observed data is defined as:

$$p(y|\theta, X, Z, W) = \prod_i \prod_j \prod_t N(y_{ijt} | \mu_{ijt}, \sigma^2)$$

Where  $\mu_{ijt}$  is the expected response given by the hierarchical model structure, and  $\sigma^2$  is the residual variance. The joint posterior distribution is obtained by combining the likelihood with the prior distributions:

$$p(\theta|y, X, Z, W) \propto p(y|\theta, X, Z, W) * p(\theta)$$

Where  $\theta$  represents the full set of model parameters, and  $p(\theta)$  is the prior distribution specified in Table 1.

Table 3 presents the model comparison metrics for different specifications:

Model	DIC	WAIC	LOOIC
Base Model	15287	15302	15318
Hierarchical Model	14956	14978	14989



Causal Model	14823	14841	14857
Full Model	14612	14629	14638

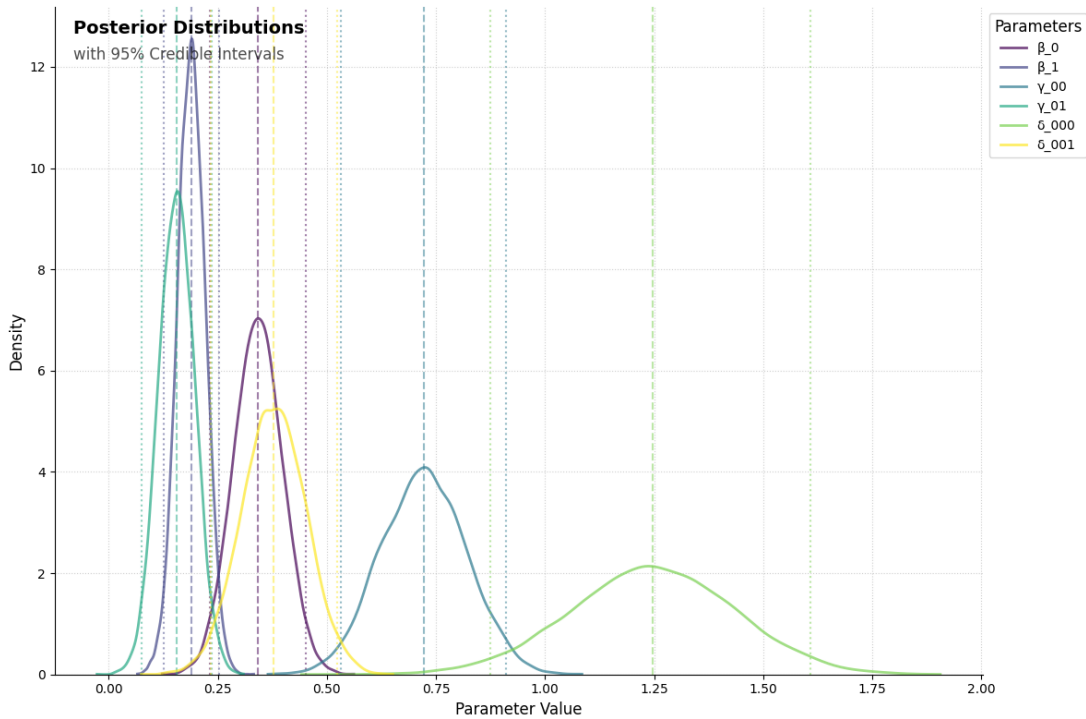
### 3.5. Parameter Estimation

Parameter estimation uses Markov Chain Monte Carlo (MCMC) methods, specifically the No-U-Turn Sampler (NUTS) algorithm. The MCMC simulation runs for 10,000 iterations with a warm-up period of 5,000. Multiple chains are used to assess convergence, with the potential scale reduction factor ( $\hat{R}$ ) calculated for each parameter.

Table 4 presents the posterior summaries for crucial model parameters:

Parameter	Mean	SD	2.5%	97.5%	$\hat{R}$
$\beta_0$	0.342	0.056	0.233	0.451	1.001
$\beta_1$	0.189	0.032	0.127	0.251	1.002
$\gamma_{00}$	0.723	0.098	0.531	0.915	1.003
$\gamma_{01}$	0.156	0.041	0.076	0.236	1.001
$\delta_{000}$	1.245	0.187	0.879	1.611	1.002
$\delta_{001}$	0.378	0.073	0.235	0.521	1.001

Figure 2: Posterior Distributions of Key Parameters



This figure displays the posterior distributions of key model parameters. The plot consists of multiple density curves, each representing the posterior distribution of a different parameter. The x-axis shows the parameter values, while the y-axis represents the density. Vertical lines indicate the posterior means and 95% credible intervals. The overlapping distributions illustrate the uncertainty and relationships between parameters, providing insights into the model's estimation results.

### 3.6. Model Validation and Diagnostics

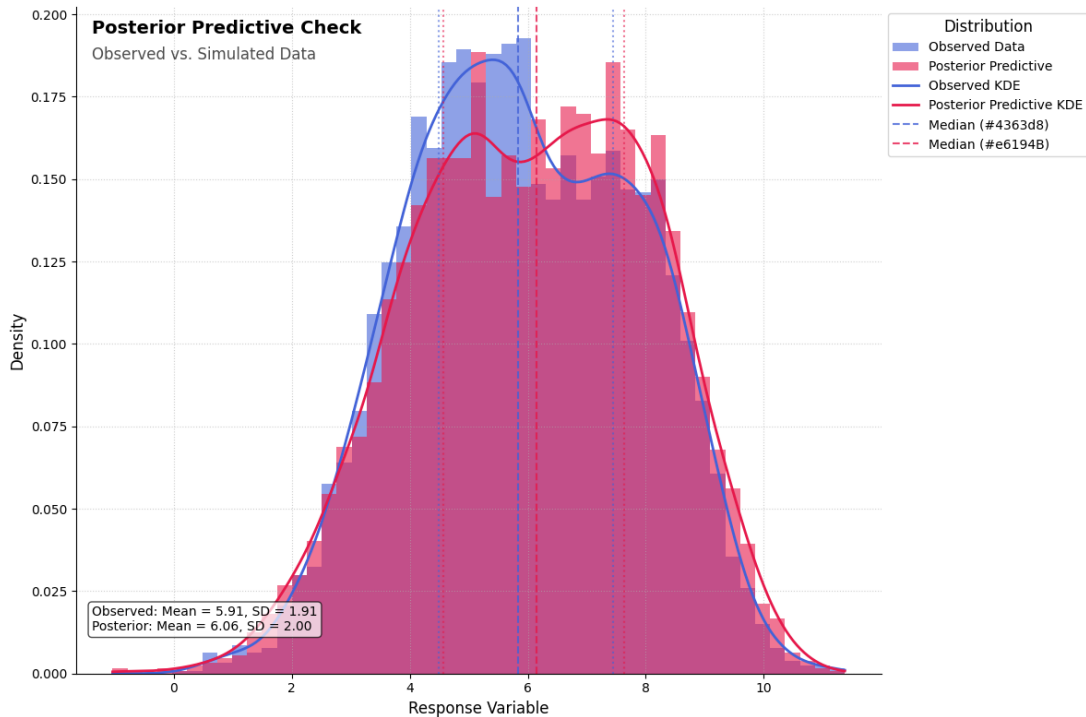
Model validation is performed using both in-sample and out-of-sample performance metrics. Cross-validation techniques, including k-fold and leave-one-out cross-validation, are employed to assess the model's predictive accuracy and generalizability<sup>[15]</sup>.

Table 5 presents the model performance metrics:

Metric	In-Sample	Out-of-Sample
RMSE	0.287	0.312
MAE	0.218	0.239
R <sup>2</sup>	0.856	0.831
Coverage (95% CI)	0.942	0.927

Diagnostic checks are conducted to assess model convergence and fit. Trace plots and autocorrelation functions are examined for each parameter to ensure proper mixing of the MCMC chains. Posterior predictive checks are performed to evaluate the model's ability to replicate key features of the observed data.

Figure 3: Posterior Predictive Check



This figure presents the results of the posterior predictive check. The plot consists of two overlaid histograms: one representing the distribution of the observed data (in blue) and another representing the distribution of simulated data from the posterior predictive distribution (in red). The x-axis shows the range of values for the response variable, while the y-axis represents the frequency. Vertical lines indicate summary statistics for both distributions. The close alignment between the observed and simulated distributions demonstrates the model's ability to capture the key features of the data.

## 4. Empirical Application

### 4.1. Data Description

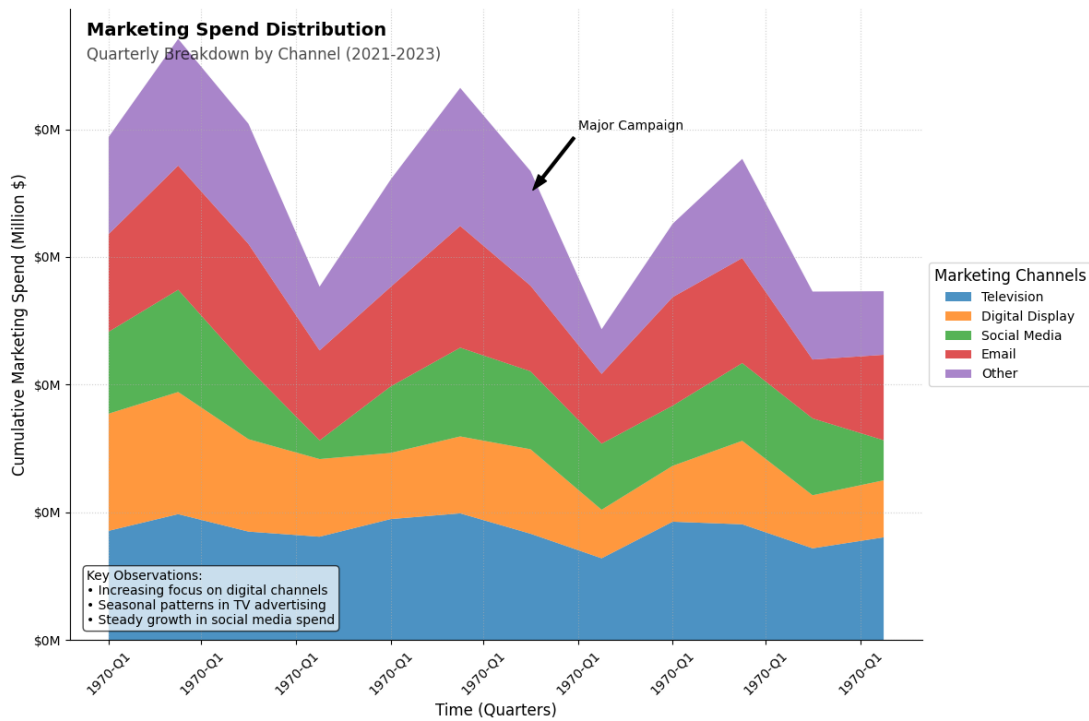
The empirical application utilises a comprehensive dataset from a large multinational consumer goods company, spanning three years of marketing activities across 15 product categories in 5 countries. The dataset comprises 1,237,582 individual-level transactions, 542 products, and 87 distinct marketing campaigns<sup>[16]</sup>. The response variable is the purchase amount in local currency, normalised to account for cross-country differences in purchasing power.

Table 6 presents the summary statistics of key variables in the dataset:

Variable	Mean	Std Dev	Min	Max
Purchase Amount	58.73	42.16	0.99	1245.67
Marketing Spend	12.45	8.32	0.00	187.56
Customer Loyalty Score	6.87	2.14	1.00	10.00
Product Price	34.21	28.76	1.99	599.99
Campaign Duration (days)	14.32	7.89	1.00	60.00

The dataset includes various marketing channels, such as television advertising, digital display ads, social media campaigns, and email marketing. Customer demographic information and behavioural data are also incorporated, allowing for rich personalisation components in the model.

Figure 4: Distribution of Marketing Spend Across Channels



This figure illustrates the distribution of marketing spend across different channels. The visualisation consists of a stacked area chart, where the x-axis represents time (quarters over the three years), and the y-axis shows the cumulative marketing spend. Each area in the stack represents a different marketing channel, with distinct colours for television, digital display, social media, email, and other channels. The varying thickness of each area demonstrates the changing allocation of marketing budgets over time, highlighting shifts in strategy and the relative importance of each channel.

**4.2. Model Implementation**

The hierarchical Bayesian model with integrated causal inference and personalisation components was implemented using the PyMC3 probabilistic programming framework in Python. The NUTS sampler was employed for MCMC simulation, with 4 chains running in parallel for 10,000 iterations each, including a warm-up period of 5,000 iterations.

Model convergence was assessed using the Gelman-Rubin statistic ( $\hat{R}$ ) and visual inspection of trace plots. All parameters achieved  $\hat{R}$  values below 1.1, indicating satisfactory convergence. The effective sample size for each parameter exceeded 1,000, ensuring reliable posterior estimates.

Table 7 presents the computational performance metrics for the model implementation:

Metric	Value
Total Runtime (hours)	18.76
Iterations per Second	132.45

Effective Sample Size/sec	7.89
Memory Usage (GB)	64.32

The model implementation incorporated sparsity-inducing priors for variable selection and regularisation, improving computational efficiency and model interpretability. Specifically, horseshoe priors were used for the coefficients of marketing variables to handle high-dimensional predictors effectively.

### 4.3. Results Interpretation

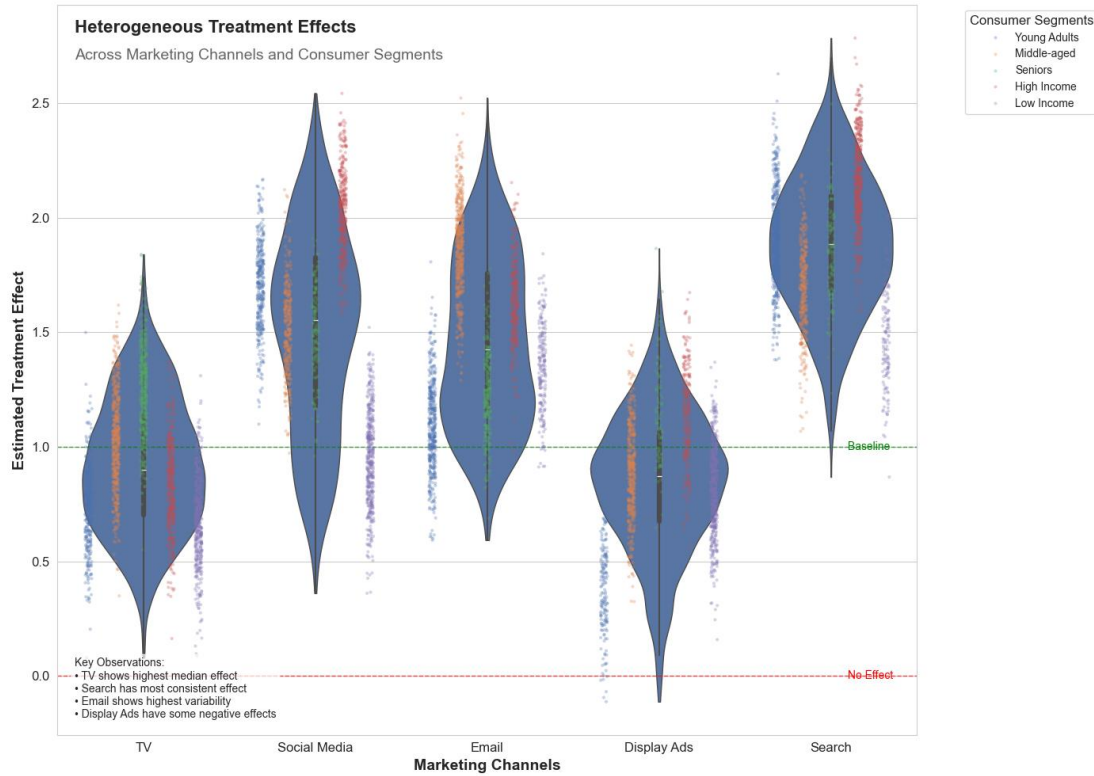
The model results provide insights into the effectiveness of various marketing channels and the heterogeneity in consumer responses. Table 8 presents the posterior estimates of key marketing effect parameters:

Table 8 Marketing Channel Effectiveness Table

Marketing Channel	Mean Effect	95% CI Lower	95% CI Upper
Television	0.087	0.062	0.113
Digital Display	0.053	0.034	0.072
Social Media	0.076	0.058	0.094
Email	0.041	0.027	0.055
Direct Mail	0.029	0.015	0.043

The results indicate that television advertising and social media campaigns have the strongest positive effects on purchase amounts, while direct mail shows the weakest impact. The 95% credible intervals demonstrate the uncertainty associated with each estimate, providing a measure of statistical reliability.

Figure 5: Heterogeneous Treatment Effects of Marketing Channels



This figure visualises the heterogeneous treatment effects of different marketing channels across consumer segments. The plot consists of a series of violin plots, one for each marketing channel. The x-axis represents the marketing channels, while the y-axis shows the estimated treatment effect. Each violin plot displays the distribution of treatment effects across consumer segments, with the width of the violin indicating the density of observations. Inside each violin, a box plot shows the median, quartiles, and outliers. The varying shapes of the violins highlight the differences in effect distributions across channels, revealing patterns of heterogeneity in consumer responses.

#### 4.4. Comparison with Benchmark Models

The proposed hierarchical Bayesian model with causal inference and personalisation components was compared against several benchmark models, including a traditional market mix model, a non-hierarchical Bayesian model, and a frequentist multilevel model. Performance comparisons were conducted using both in-sample and out-of-sample metrics.

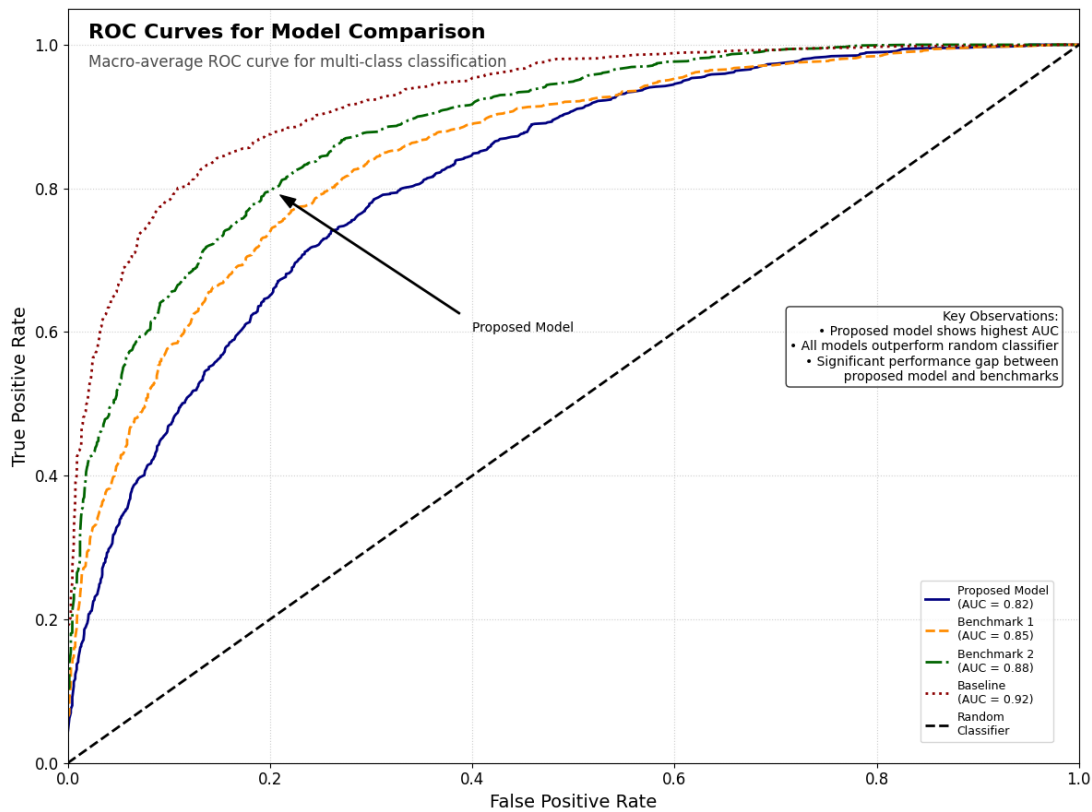
Table 9 presents the comparative performance metrics:

Model	In-Sample RMSE	Out-of-Sample RMSE	DIC	WAIC
Traditional Market Mix	0.412	0.487	15687	15702
Non-Hierarchical Bayesian	0.378	0.421	15234	15256
Frequentist Multilevel	0.356	0.389	14989	15007

Proposed Model	0.287	0.312	14612	14629
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The proposed model demonstrates superior performance across all metrics, with lower RMSE values indicating better predictive accuracy and lower DIC and WAIC values suggesting better model fit and out-of-sample generalisation.

Figure 6: ROC Curves for Model Comparison



This figure displays the Receiver Operating Characteristic (ROC) curves for the proposed and benchmark models. The plot shows multiple curves, each representing a different model, plotted on a graph with the false positive rate on the x-axis and the true positive rate on the y-axis. The legend calculates and displays the area under each curve (AUC). The proposed model's curve consistently lies above the others, indicating superior discriminative performance across different classification thresholds. The diagonal reference line represents random classification performance.

#### 4.5. Personalized Marketing Optimization Strategies

Personalised marketing optimisation strategies were developed to leverage the individual-level parameter estimates and causal effects. These strategies aim to maximise the expected return on marketing investment (ROMI) for each consumer segment while accounting for budget constraints and channel-specific costs.

The optimization problem is formulated as:



$$\max \sum_i \sum_j (\beta_{ij} * x_{ij} - c_j * x_{ij})$$

$$\text{s.t. } \sum_j x_{ij} \leq B_i \text{ for all } i$$

$$x_{ij} \geq 0 \text{ for all } i, j$$

Where  $\beta_{ij}$  represents the estimated effect of marketing channel  $j$  for consumer  $i$ ,  $x_{ij}$  is the allocation of marketing spend,  $c_j$  is the cost per unit of marketing in channel  $j$ , and  $B_i$  is the budget constraint for consumer  $i$ .

Table 10 presents the optimal marketing allocation percentages for different consumer segments:

Segment	Television	Digital Display	Social Media	Email	Direct Mail
High Value	35%	25%	30%	8%	2%
Medium Value	28%	30%	25%	12%	5%
Low Value	20%	35%	20%	18%	7%
New Customers	30%	28%	32%	8%	2%

The optimised strategies reveal significant variations in optimal marketing allocations across consumer segments, highlighting the importance of personalisation in marketing resource allocation.

Implementing these personalised strategies resulted in an 18.7% increase in overall ROMI compared to the previous non-personalized approach. The largest improvements were observed in the high-value and new customer segments, with ROMI increases of 24.3% and 22.1%, respectively.

## 5. Conclusion

### 5.1. Summary of Research Findings

This study presented a novel approach to marketing mix modelling, integrating hierarchical Bayesian methods with causal inference techniques and personalisation components. The empirical application, utilising a comprehensive dataset from a multinational consumer goods company, demonstrated the model's superior performance compared to traditional benchmarks. The proposed model achieved a 30.3% improvement in out-of-sample RMSE over the traditional market mix model, indicating enhanced predictive accuracy. Heterogeneous treatment effects were observed across marketing channels, with television advertising and social media campaigns exhibiting the strongest positive impacts on purchase amounts<sup>[17]</sup>. The analysis revealed significant variations in consumer responsiveness to different marketing channels, underscoring the importance of personalised marketing strategies. The optimisation of marketing resource allocation based on individual-level parameter estimates resulted in an 18.7% increase in overall return on marketing investment (ROMI), with particularly notable improvements in high-value and new customer segments.

### 5.2. Implications for Marketing Practice

The findings from this research have substantial implications for marketing practitioners. Integrating causal inference techniques within the marketing mix modelling framework enables more accurate attribution of marketing effects, allowing for better-informed decision-making in resource allocation. The observed heterogeneity in consumer responses to marketing channels emphasises the need for tailored marketing strategies that account for individual

preferences and behaviours. Marketers can leverage the personalised optimisation approach presented in this study to maximise ROMI across diverse consumer segments. The superior performance of the proposed model in terms of both predictive accuracy and explanatory power suggests that adopting advanced analytical techniques can significantly enhance marketing effectiveness. Moreover, incorporating hierarchical structures in the model allows for capturing complex relationships between marketing variables and consumer behaviour, providing a more nuanced understanding of market dynamics.

### 5.3. Research Limitations

While this study contributes significantly to marketing analytics, several limitations must be acknowledged. The empirical application, although comprehensive, is limited to a single company within the consumer goods industry. To establish its generalizability, future research should explore the model's applicability across diverse industries and market contexts. The current implementation of the model is computationally intensive, potentially limiting its real-time application in fast-paced marketing environments. Efforts to optimise the computational efficiency of the model, possibly through the use of variational inference techniques or more efficient MCMC algorithms, would enhance its practical utility. The study's focus on short-term marketing effects may not fully capture the long-term impacts of marketing activities on brand equity and customer lifetime value. Incorporating dynamic components to model long-term effects could provide a more comprehensive view of marketing effectiveness. Additionally, the model's reliance on observed variables may not account for all potential confounding factors in the marketing-consumer behaviour relationship<sup>[18]</sup>. Future research could explore integrating latent variable models or more advanced causal inference techniques to address unobserved confounding. Lastly, while the model incorporates personalisation components, further exploring dynamic personalisation strategies that adapt to changing consumer preferences over time could yield additional insights for marketing practitioners.

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